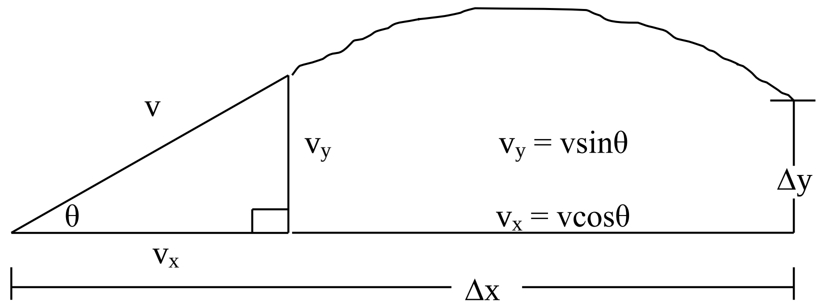
Physics Lecture #11: Projectiles Launched at an Angle, Part 1

When a projectile is launched at an angle, it initially moves diagonally. Thus, it is moving both vertically and horizontally. If we know the initial diagonal velocity v, we can find the horizontal velocity v_x and the initial vertical velocity, v_y. We can also find the horizontal distance, Δx, and the vertical distance, Δy.



A soccer ball is kicked and moves at an angle of 35^o with the horizontal at an initial velocity of 6.4 m/s. Find the initial horizontal and vertical velocities. Find the horizontal and vertical distances covered after 0.40 seconds.

Answer

$v = 6.4$ m/s $\quad \theta = 35^o \quad v_x = ? \quad v_y = ?$

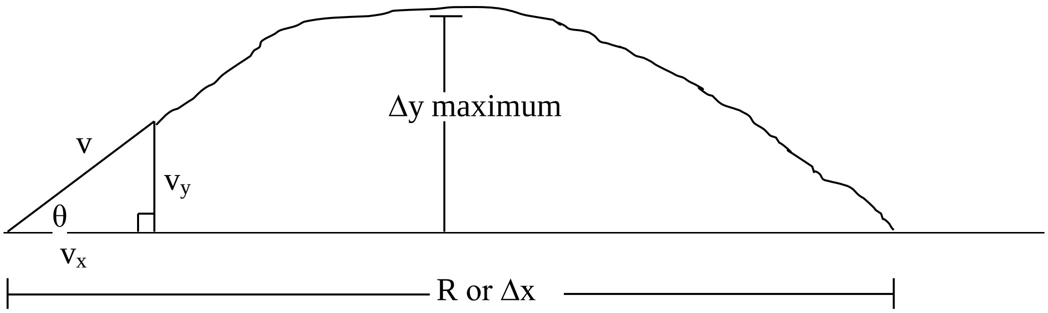
$v_x = v\cos\theta$	$v_y = v\sin\theta$
$v_x = 6.4(\cos 35)$	$v_y = 6.4(\sin 35)$
$v_x = 5.24$ or 5.2 m/s	$v_y = 3.67$ or 3.7 m/s

The horizontal velocity, v_x stays constant. However, the vertical velocity, v_y, changes over time due to the influence of gravity. Gravity accelerates an object at downward at 9.81 m/s^2. We'll use -9.81 m/s^2. Notice that this is a negative number. Remember that we use a negative value for acceleration when an object slows down and/or changes direction. An object launched at an angle will go up, slow its ascent, reach a peak, then change direction and move down.

To find the horizontal and vertical distances covered, we use

$\Delta x = v_x t$	$\Delta y = v_y t + \frac{1}{2} at^2$ where $a = -9.81$ m/s^2
$\Delta x = 5.2(0.40)$	$\Delta y = 3.7(0.4) + \frac{1}{2}(-9.81)(0.4)^2$
$\Delta x = 2.08$ or 2.1 m horizontal distance	$\Delta y = 1.48 - 0.7848$
	$\Delta y = 0.6952$ or 0.70 m vertical distance

If a projectile fired at an angle lands at the same height from which it was launched, we can use the range formula to find the horizontal distance, R. Knowing R, we can calculate the time the projectile is airborne, and the maximum height reached by the object.



$$R = \frac{v^2 \sin 2\theta}{g} \qquad g = 9.81 \text{ m/s}^2$$

A cannon ball is shot at an angle of 52^o with the horizontal. It leaves at a velocity of 33 m/s. Find the horizontal distance it travels. Find the time it is in the air, and the maximum height it reaches.

Answer

Before we use the range formula, we'll solve for v_x and v_y, then use these values later on.

$v_x = v\cos\theta$	$v_y = v\sin\theta$
$v_x = 33\cos 52$	$v_y = 33\sin 52$
$v_x = 20.32$ m/s	$v_y = 26.00$ m/s

$$R = \frac{v^2 \sin 2\theta}{g} = \frac{33^2 \sin 2(52)}{9.81} = \frac{1089 \sin 104}{9.81} = \frac{1089(0.9702)}{9.81} = 107.7 \text{ or } 1.1 \times 10^2 \text{ m}$$

The cannon ball travels a horizontal distance of 107.7 m. Since we know the horizontal velocity, v_x, we use $v_x = \Delta x/t$ to find the time it is airborne.

$$v_x = \frac{\Delta x}{t} \Rightarrow 20.32 = \frac{107.7}{t} \Rightarrow t = \frac{107.7}{20.32} \Rightarrow t = 5.3 \text{ seconds}$$

When the ball reaches maximum height, its vertical velocity is zero. Thus, we can use $v_y^2 = 2a\Delta y$ to solve for Δy.

$$v_y^2 = 2a\Delta y \Rightarrow 26^2 = 2(9.81)\Delta y \Rightarrow \Delta y = \frac{26^2}{2(9.81)} \Rightarrow \Delta y = 34 \text{ m maximum height}$$