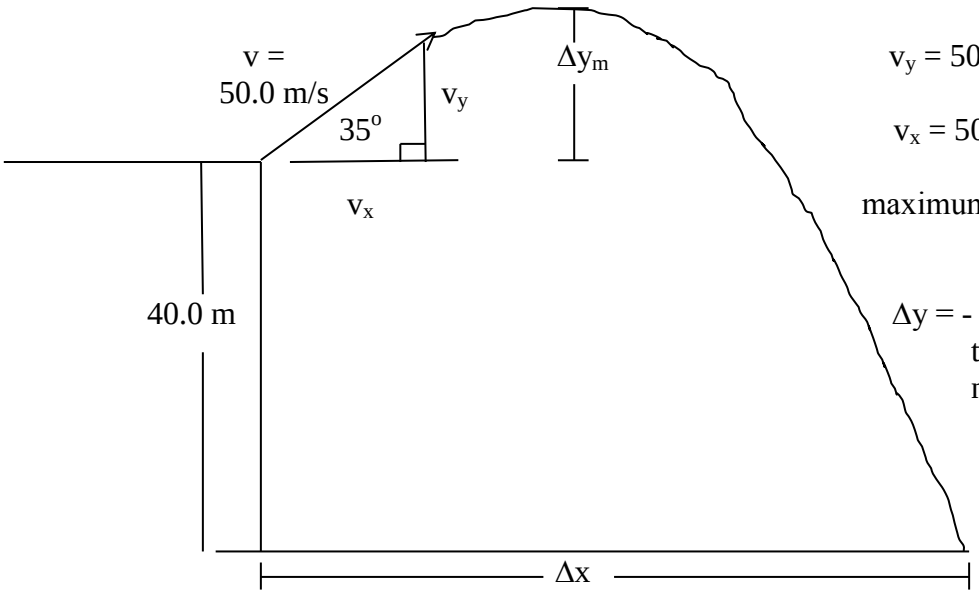
Physics Lecture #12: Projectiles Launched at an Angle, Part 2

There are two types of projectile problems that are hard to solve, so I'm putting them in a separate lecture.

Problem #1: A cannon ball is shot from the edge of a 40.0 m tall cliff at a speed of 50.0 m/s at an angle of 35^o with the horizontal. Find the maximum height it reaches. Find the time it is airborne and the horizontal distance it travels before hitting the ground.

Answer



$v_y = 50\sin 35$

$v_x = 50\cos 35$

maximum height = $40.0 + \Delta y_m$

$\Delta y = -40.0$ m (notice that this is a negative number)

$v_y = 50\sin 35$	$v_x = 50\cos 35$
$v_y = 50(0.5735)$	$v_x = 50(0.8191)$
$v_y = 28.6788$	$v_x = 40.9550$

When the cannon ball reaches the maximum peak, its vertical velocity is zero. Thus, we can use $v_y^2 = 2a\Delta y_m$ to find Δy_m.

$v_y^2 = 2a\Delta y_m$

$28.6788^2 = 2(9.81)\Delta y_m$

$$\Delta y_m = \frac{28.6788^2}{2(9.81)}$$

$\Delta y_m = 41.92$

Maximum height = 40.0 + 41.92 = 81.92 or 82 m

To find the time the ball is airborne, we use $\Delta y = v_y t + \frac{1}{2} at^2$. We set Δy equal to *negative* 40.0 m because the ball ends up *below* its original height. Also, we set "a" equal to *negative* 9.81 m/s^2 because the ball's vertical velocity slows down and changes direction – it first goes up, slows down, stops, then goes down.

$\Delta y = v_y t + \frac{1}{2} at^2$

$-40.0 = 28.6788t + \frac{1}{2}(-9.81)t^2$

$\frac{1}{2}(9.81)t^2 - 28.6788t - 40.0 = 0$

$4.905t^2 - 28.6788t - 40.0 = 0$ Divide both sides by 4.905

$t^2 - 5.8468t - 8.1549 = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-5.8468) \pm \sqrt{5.8468^2 - 4(1)(-8.1549)}}{2(1)}$$

$$t = \frac{5.8468 \pm 8.1734}{2}$$

t = -1.1633 or 7.0101. We pick the positive value for time, so t = 7.01 or 7.0 seconds airborne.

To find horizontal distance, we use $\Delta x = v_x t$

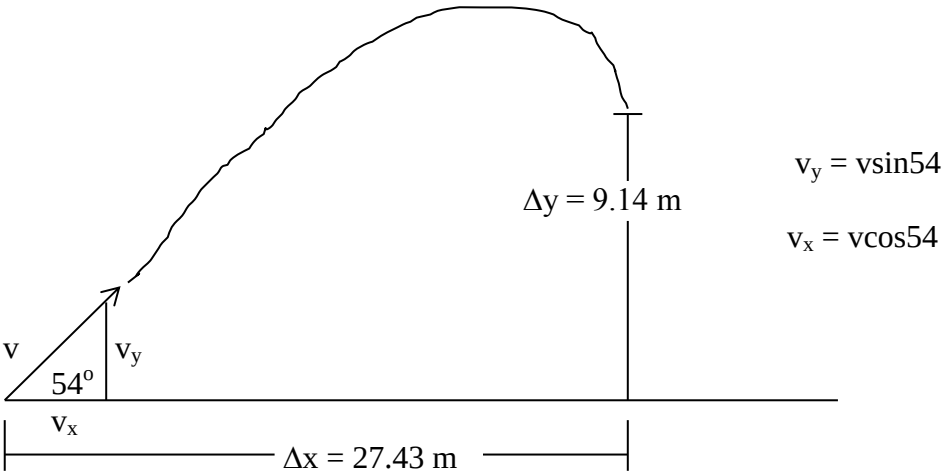
$\Delta x = v_x t$

$\Delta x = 40.9550(7.01)$

$\Delta x = 287.09$ or 2.9×10^2 m horizontal distance.

Problem #2: A football player attempts to kick a field goal from 27.43 m away. The horizontal bar of the goal post is 9.14 m high. He kicks the ball, which initially moves at an angle of 54^o from the horizontal. The ball lands on the horizontal bar. What was the initial velocity of the ball?

Answer



To find velocity, we need the time, t. We'll define v in terms of t, and substitute it into $\Delta y = v_y t + \frac{1}{2} at^2$ and solve for t. Once we have t, we can use it to find v.

$\Delta x = v_x t$

$27.43 = (v\cos 54)t$

$27.43 = v(0.5877)t$

$$v = \frac{27.43}{(0.5877)t}$$

$$v = \frac{46.67}{t}$$

$\Delta y = v_y t + \frac{1}{2} at^2$

$9.14 = (v\sin 54)t + \frac{1}{2}(-9.81)t^2$

$9.14 = \frac{46.67}{t}(0.8090)t - 4.905t^2$

$9.14 = 46.67(0.8090) - 4.905t^2$

$9.14 = 37.7560 - 4.905t^2$

$4.905t^2 = 37.7560 - 9.14$

$4.905t^2 = 28.616$

$t^2 = 5.834$

$t = 2.4153$ seconds

$v = \frac{46.67}{t}$

$v = \frac{46.67}{2.4153}$

$v = 19.32$ or 19 m/s