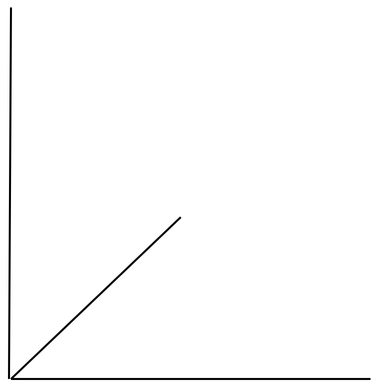
**Physics Lecture #4: Calculating Distance from Initial and Final Velocity**

**Acceleration is the change in velocity over time, or $a = \Delta v/t$. Uniform acceleration means that the velocity is changing at a constant rate. For example, if a car was increasing its speed at a constant rate, a velocity – time graph would be a straight line like this:**

**velocity**

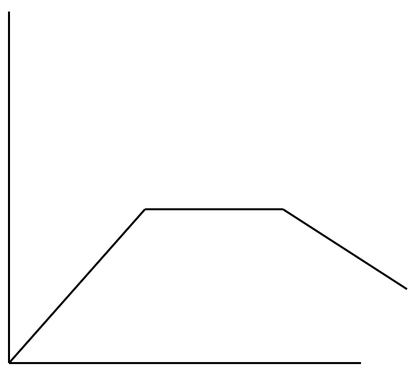


**uniform acceleration - constant slope**

**time**

**If, however, the car sped up, then maintained a constant speed, and then slowed down, the velocity – time graph would look like this:**

**velocity**



**non-uniform acceleration – changing slope**

**time**

**If an object is undergoing non-uniform acceleration, its speed is not changing at a constant rate. Its speed varies between increasing, slowing down, or remaining constant.**

**If an object is undergoing uniform acceleration, we can calculate the distance it covers over an elapsed time period.**

**One simple way to calculate the distance covered is to use the formula for average velocity:**

$$v_{avg} = \frac{distance}{time} \quad \text{or} \quad v_{avg} = \frac{\Delta x}{t}$$

**Find the distance covered by a car moving at an average velocity of 12 m/s for 4.0 seconds.**

$$v_{avg} = \frac{\Delta x}{t}$$

$$12 = \frac{\Delta x}{4.0}$$

$$\Delta x = 12\ (4.0) = 48\ \text{m}$$

**Suppose a car is undergoing uniform acceleration. Over an 8.0 second period, its velocity changes from 15 m/s to 31 m/s. How would we find the average velocity and distance covered?**

**If we want the average of two numbers, we add them together and divide by 2. Thus, the average velocity of an object undergoing uniform acceleration can also be expressed as**

$$v_{avg} = \frac{v_i + v_f}{2}$$

**Where $v_i$ = initial velocity and $v_f$ = final velocity.**

**If the velocity of an object increased from 15 m/s to 31 m/s, its average velocity would be**

$$v_{avg} = \frac{v_i + v_f}{2} = \frac{15 + 31}{2} = 23\ \text{m/s}$$

**Knowing the average velocity, we can now calculate the distance covered over 8.0 seconds.**

$$v_{avg} = \frac{\Delta x}{t}$$

$$23 = \frac{\Delta x}{8.0}$$

$$\Delta x = 23\ (8.0) = 184\ \text{m or } 1.8 \times 10^2\ \text{m}$$

**If we substitute $(v_i + v_f)/2$ in place of $v_{avg}$, we can rearrange $v_{avg} = \Delta x/t$ to solve for $\Delta x$.**

$$v_{avg} = \frac{\Delta x}{t}$$

$$\frac{v_i + v_f}{2} = \frac{\Delta x}{t}$$ **Cross multiply, and you get**

$$2\,\Delta x = (v_i + v_f)\,t$$

$$\Delta x = \frac{(v_i + v_f)\,t}{2}$$ **Works for an object at uniform acceleration**

**We can solve the previous problem more quickly. Find the distance covered by a car that increases its velocity from 15 m/s to 31 m/s over an 8.0 second period.**

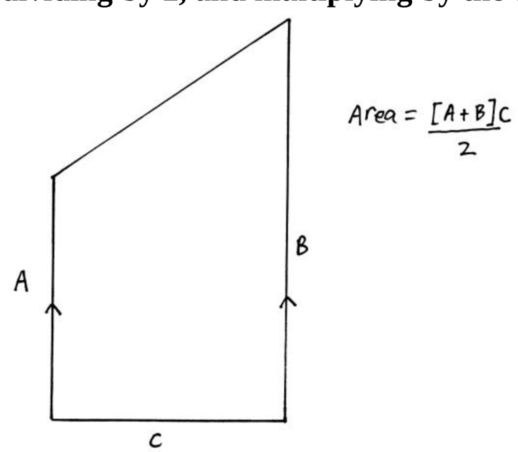
$$\Delta x = \frac{(v_i + v_f)\,t}{2}$$

$$\Delta x = \frac{(15 + 31)\,8.0}{2} = 184\text{ m or } 1.8 \times 10^2\text{ m}$$

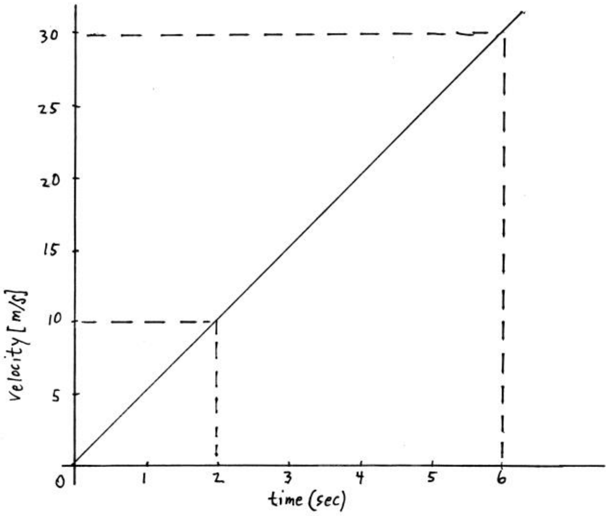
**The distance covered by an accelerating object can also be found by examining a velocity – time graph of the object's motion. More specifically,**

***The area under the line of a velocity – time graph gives the distance covered.***

**The area under a velocity – time graph can be found by inscribing a trapezoid under the line. Remember that a trapezoid is a 4-sided object that has at least two sides that are parallel. The area of the trapezoid is found by adding the lengths of the parallel sides, dividing by 2, and multiplying by the distance between the parallel sides.**



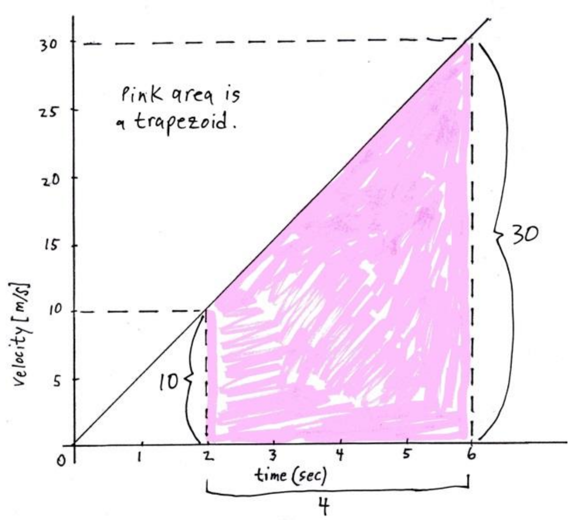
**The graph below shows an object moving at increasing velocity. Find the distance covered from 2 seconds to 6 seconds.**




**Reading the graph, we see that $v_i = 10$, $v_f = 30$, and $t = 6 - 2 = 4$. So,**

$$\Delta x = \frac{(v_i + v_f)\,t}{2} = \frac{(10 + 30)\,4}{2} = 80 \text{ m}$$

**If we color the area under the line from 2 to 6 seconds, we get a trapezoid.**




**The area of the trapezoid is $\frac{(10 + 30)\,4}{2} = 80$**

**This is the same calculation used to determine the distance covered. Thus, the area under the line and the distance covered are the same.**