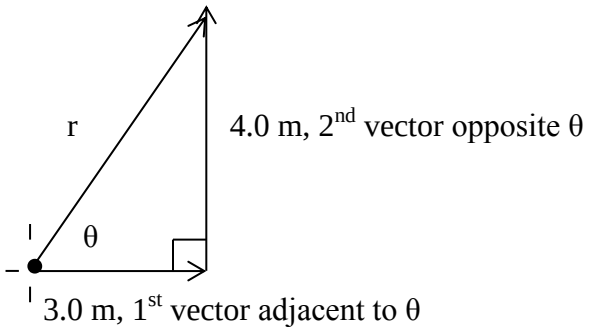
Physics Lecture #8: Adding Perpendicular Vectors

Previously we added vectors by drawing pictures. This time we'll draw a sketch of vectors being added together and use trigonometry to find the size and direction of the resultant.

A car moves 3.0 m east, then 4.0 m north. Find the direction and displacement from the starting point.

Answer

First, sketch a picture of the two vectors. Draw the first vector and put a big dot at its tail. Add dotted lines at the tail of the first vector to indicate that it is the origin of a graph. Then draw the 2^{nd} vector, placing its tail at the head of the first vector. Finally, draw a line from the tail of the first vector to the head of the 2^{nd} vector.



Notice that we've drawn a right triangle. The resultant, r, is the hypotenuse of the triangle. We can use the Pythagorean Theorem to solve for r.

$r^2 = (3.0)^2 + (4.0)^2$

$r^2 = 9.0 + 16$

$r^2 = 25$

$r = 5.0$ m

We can use the tangent function to find the direction (θ) of the resultant.

$\tan\theta$ = opposite/adjacent

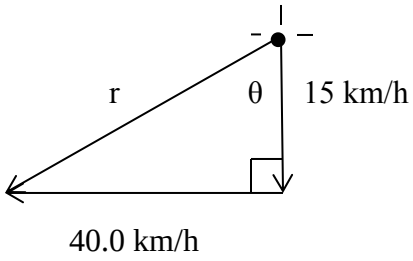
$\tan\theta = 4/3$

$\tan\theta = 1.3333$ Use $\theta = \tan^{-1}(1.3333)$

$\theta = 53^o$ north of east

A bird flies south at 15 km/h with respect to the air. The wind blows at 40.0 km/h due west with respect to the ground. Find the velocity of the bird (magnitude and direction)

Answer



$r^2 = (15)^2 + (40.0)^2$

$r^2 = 225 + 1600$

$r^2 = 1825$

$r = 42.7$ or 43 km/h

$\tan\theta$ = opposite/adjacent

$\tan\theta = 40.0/15$

$\tan\theta = 2.6666$

$\theta = \tan^{-1}(2.6666)$

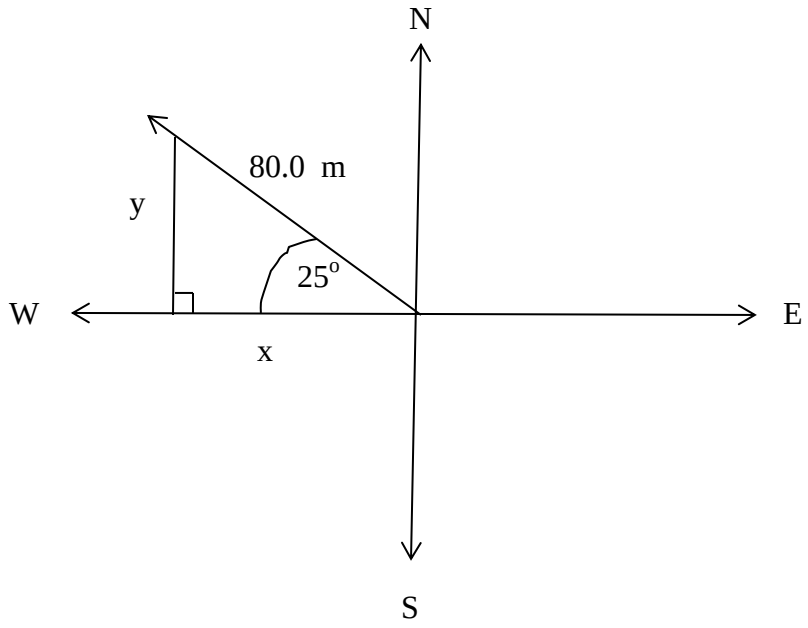
$\theta = 69^o$ west of south

In some cases, we're given a diagonal vector, and we want to know the vertical and horizontal components of the vector. In this case, we use sine and cosine to find vertical and horizontal components.

A cow wanders 80.0 m 25^o north of west. How far north has it traveled? How far west has it traveled?

Answer

Draw an x and y axis and add compass points. Draw the diagonal vector from the origin of the graph. Add a line to create a right triangle so that the diagonal vector is the hypotenuse.



Side y of the right triangle tells us how far north the cow has traveled. We can use the sine function to solve for y. Side x of the triangle tells us how far west the cow has traveled. We can use the cosine function to solve for x.

sin 25 = opposite/hypotenuse	cos 25 = adjacent/hypotenuse
$\sin 25 = \frac{y}{80.0}$	$\cos 25 = \frac{x}{80.0}$
$0.4226 = \frac{y}{80.0}$	$0.9063 = \frac{x}{80.0}$
y = (0.4226)(80.0)	x = (0.9063)(80.0)
y = 33.8 or 34 m north	x = 72.5 or 73 m west