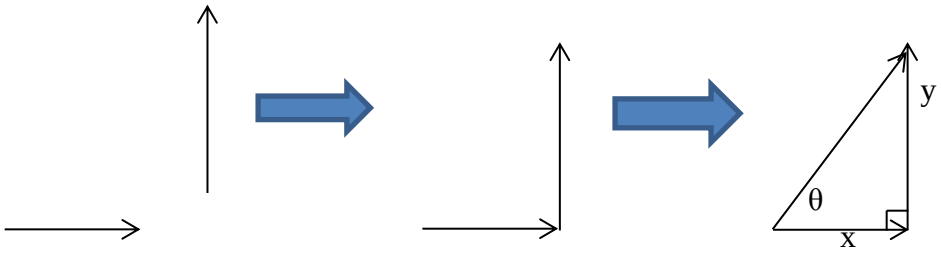
Physics Lecture #9: Adding Diagonal Vectors

If one vector is horizontal and another is vertical, the vectors are perpendicular to each other. To add perpendicular vectors, you can place the tail of the vertical vector at the head of the horizontal vector. Then you draw a line from the tail of the horizontal vector to the head of the vertical vector to get the resultant vector. You end up with a right triangle, allowing you to use trigonometry to find the hypotenuse and the angle.

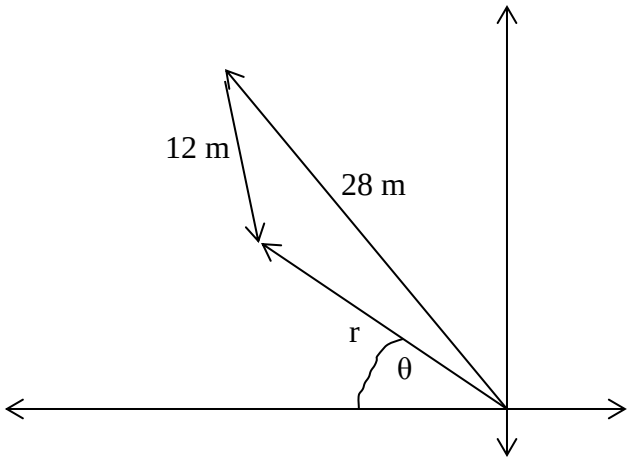


But what if we had to add two vectors that were not horizontal and vertical. How would we add two diagonal vectors?

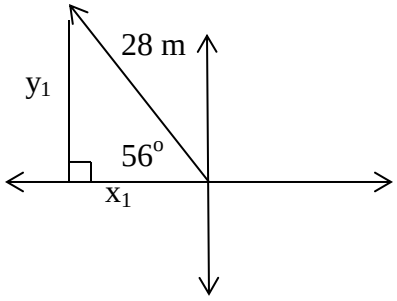
We can add diagonal vectors by resolving them into their vertical (y) and horizontal (x) components. We add up the vertical components to get y_t. We add up the horizontal components to get x_t. We now have perpendicular vectors, y_t and x_t , which we can add to find the resultant vector.

Problem
A dog walks 28 m from its home in a direction 56^o north of west. The dog then walks 12 m in a direction 75^o south of east. Find the magnitude and direction of the dog's displacement.

Answer
A sketch of the dog's path and resultant displacement looks like this:



We'll sketch each vector with its tail at the origin of a graph, then resolve them into x and y components. 28 m in a direction 56^o north of west looks like this:



$\sin 56 = \frac{y_1}{28}$ | $\cos 56 = \frac{x_1}{28}$

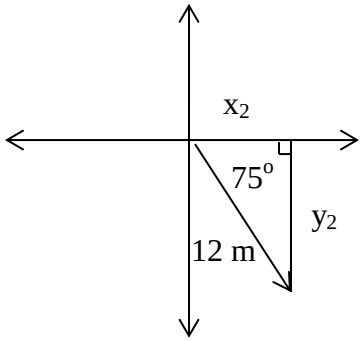
$0.8290 = \frac{y_1}{28}$ | $0.5591 = \frac{x_1}{28}$

$y_1 = (28)(0.8290)$ | $x_1 = (28)(0.5591)$

$y_1 = 23.2120$ up | $x_1 = 15.6548$ left or -15.6548

If the horizontal component points left, it has to be negative.

12 m in a direction 75^o south of east looks like this:



$\sin 75 = \frac{y_2}{12}$ | $\cos 75 = \frac{x_2}{12}$

$0.9659 = \frac{y_2}{12}$ | $0.2588 = \frac{x_2}{12}$

$y_2 = (0.9659)(12)$ | $x_2 = (0.2588)(12)$

$y_2 = 11.5908$ down or -11.5908 | $x_2 = 3.1056$

If the vertical component points down, it has to be negative.

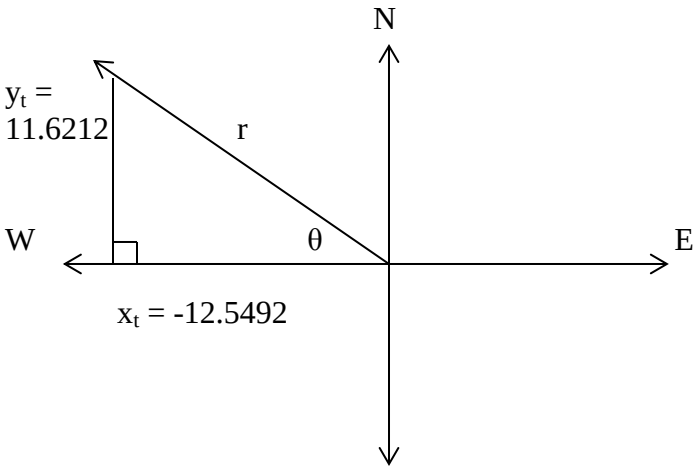
The total sum of the y components, y_t, is $y_t = y_1 + y_2$. The total sum of x components, x_t, is $x_t = x_1 + x_2$.

$y_t = y_1 + y_2$ | $x_t = x_1 + x_2$

$y_t = 23.2120 + (-11.5908)$ | $x_t = -15.6548 + 3.1056$

$y_t = 11.6212$ positive, points up | $x_t = -12.5492$ negative, points to the left

We can now determine a new vector by adding y_t and x_t.



$r^2 = (-12.5492)^2 + (11.6212)^2$

$r^2 = 157.4824 + 135.05228$

$r^2 = 292.5346$

r = 17.1036 or 17 m displacement

$\tan\theta = \frac{11.6212}{12.5492}$

$\tan\theta = 0.9260$

$\theta = \tan^{-1}(0.9260)$

$\theta = 42.799$ or 43^o north of west

A sketch of the dog's path is shown below.